

13.01.2012

- Sparse spectrum Entropy Influence Conj
- Proof of Friedgut's Thm
- Other open problems ("Gaussian" open problems)

Example Dictator: depends on one coordinate, $\sum I_i = \text{small}$

$$f(x_1, \dots, x_k, y_0, \dots, y_{2^k-1}) = y_{x_1, \dots, x_k}$$

$$= \sum_{i=0}^{2^k-1} y_i \prod_{j=1}^k \frac{1 - (-1)^{y_j} (-1)^{ij}}{2}$$

\Rightarrow Fourier expansion is of deg $\leq k+1$

$\sum I_i \leq k+1$
 e.g. If $\mathbb{P}(f=g) \geq \frac{3}{4}$ then g depends on at least 2^{k-1} coordinates.

another way to see this

$$\sum I_i(f) = \sum_{i=1}^k I_i(f) + \sum_{k+1}^{2^k+k} I_i(f) \leq k + 2^k \cdot 2^{-k} = k+1.$$

Question Is there a monotone example?

Theorem (Friedgut) Let $f: \{-1,1\}^n \rightarrow \{-1,1\}$ satisfy $\sum I_i(f) \leq k$.
 Then there exists $g: \{-1,1\}^n \rightarrow \{-1,1\}$ which depends on at most $k \cdot 2 \frac{16k}{\epsilon}$ coordinates and $\mathbb{P}(f \neq g) \leq \epsilon$.

Proof. Let $f_i = \frac{1}{2} (f(x_1, \dots, \underset{i}{1}, \dots, x_n) - f(x_1, \dots, \underset{i}{-1}, \dots, x_n))$
 $= \sum_{s: i \in s} \hat{f}(s) x_s$

By hypercontractivity

$$\|T_s f_i\|_2^2 \leq \|f_i\|_{1+s^2}^2 = I_i \frac{2}{1+s^2}$$

Let

$$A = \{i : I_i(f) < \eta\} \quad (\eta \text{ will be chosen later})$$

(g will be a function of coordinates in A^c)

Look at

$$\textcircled{1} \quad \hat{\mathbb{E}}(|S|) = \sum \hat{f}(s)^2 |S| = \sum I_i(f) \leq k$$

\Downarrow Markov

$$\hat{\mathbb{P}}(|S| \geq \frac{4k}{\epsilon}) \leq \frac{\epsilon}{4}$$

$$\hat{\mathbb{P}}(s) = \hat{f}(s)^2$$

